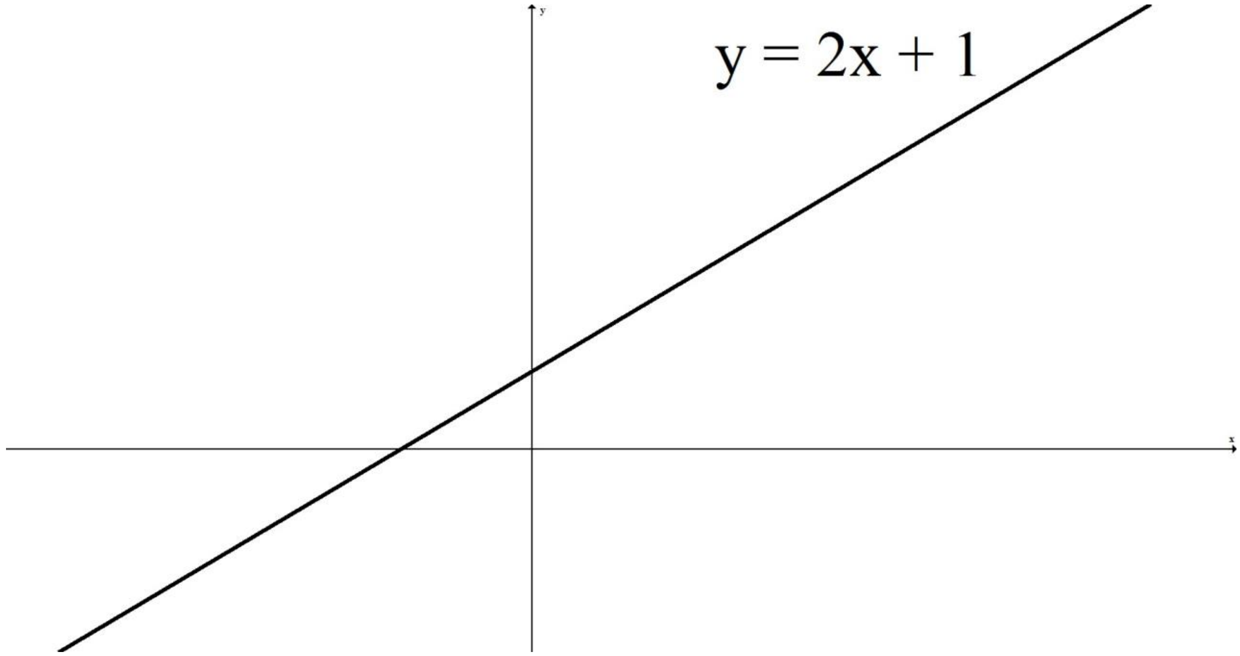
Closing Thu, Jan. 15: 12.4(1)(2)
Closing Tue, Jan. 20: 12.5(1)(2)(3)
Closing Thu, Jan. 22: 12.6

12.5 Lines/Planes in 3 Dimensions

LINES

In order to describe **lines** in 3D, we will use vectors and parametric equations. As warm up and motivation, here is an example of the idea in 2 dimensions:



Ex: Consider the line y = 2x + 1.

(a) Find a vector that is parallel to this line. Call this vector **v.**

(b) Find a vector whose head touches the line when drawn from the origin.
Call this vector $\mathbf{r_0}$**.**

(c) We can reach all other points on the line by walking along $\mathbf{r_0}$**,** then adding scale multiples of **v**.

We can use this idea to describe any line in 2 or 3 dimensions using vectors.

The equation for a line in 3D:

If we can find:

$\boldsymbol{v} = \langle a, b, c \rangle =$ a vector parallel to the line.

$\boldsymbol{r_0} = \langle x_0, y_0, z_0 \rangle =$ a position vector (which means (x_0, y_0, z_0) is a point on the line)

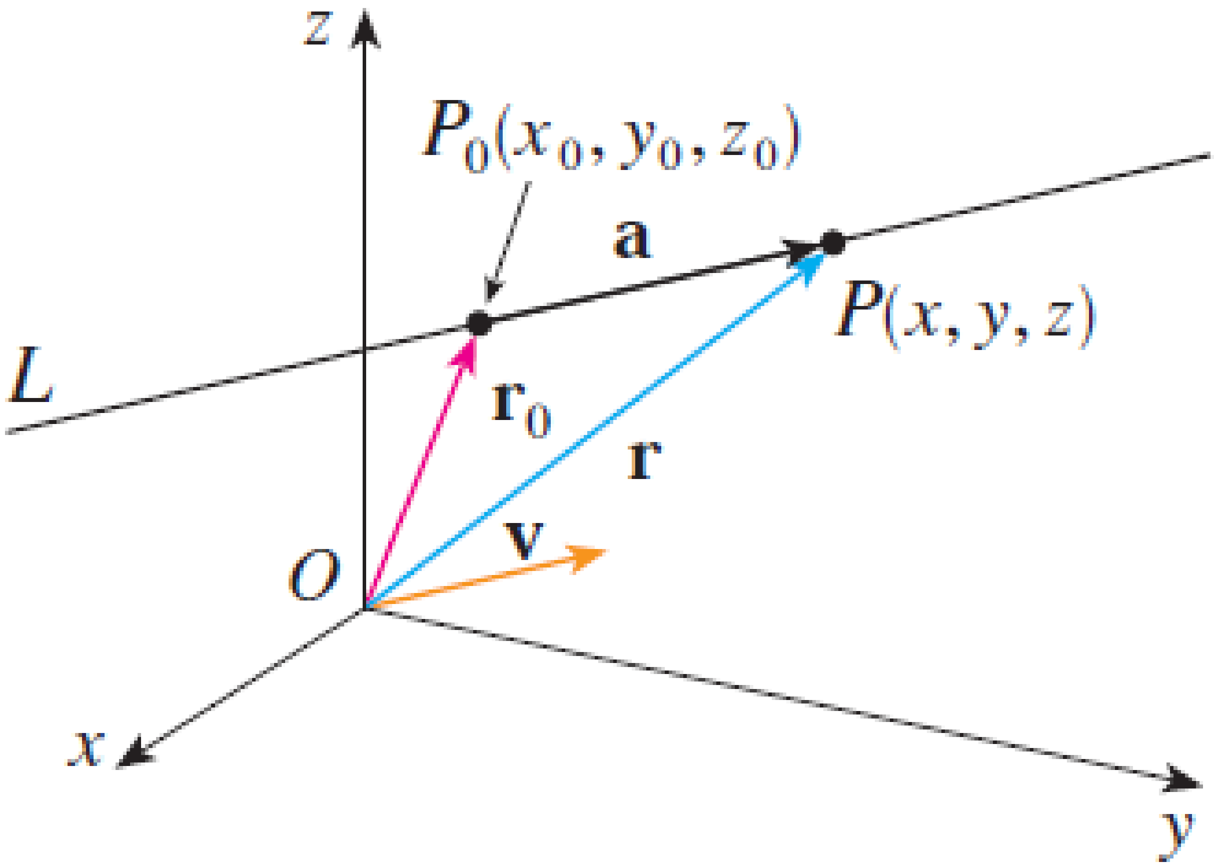
then all other points on the line can be obtained by

$$\langle x, y, z \rangle = \langle x_0, y_0, z_0 \rangle + \mathrm{t}\langle a, b, c \rangle,$$

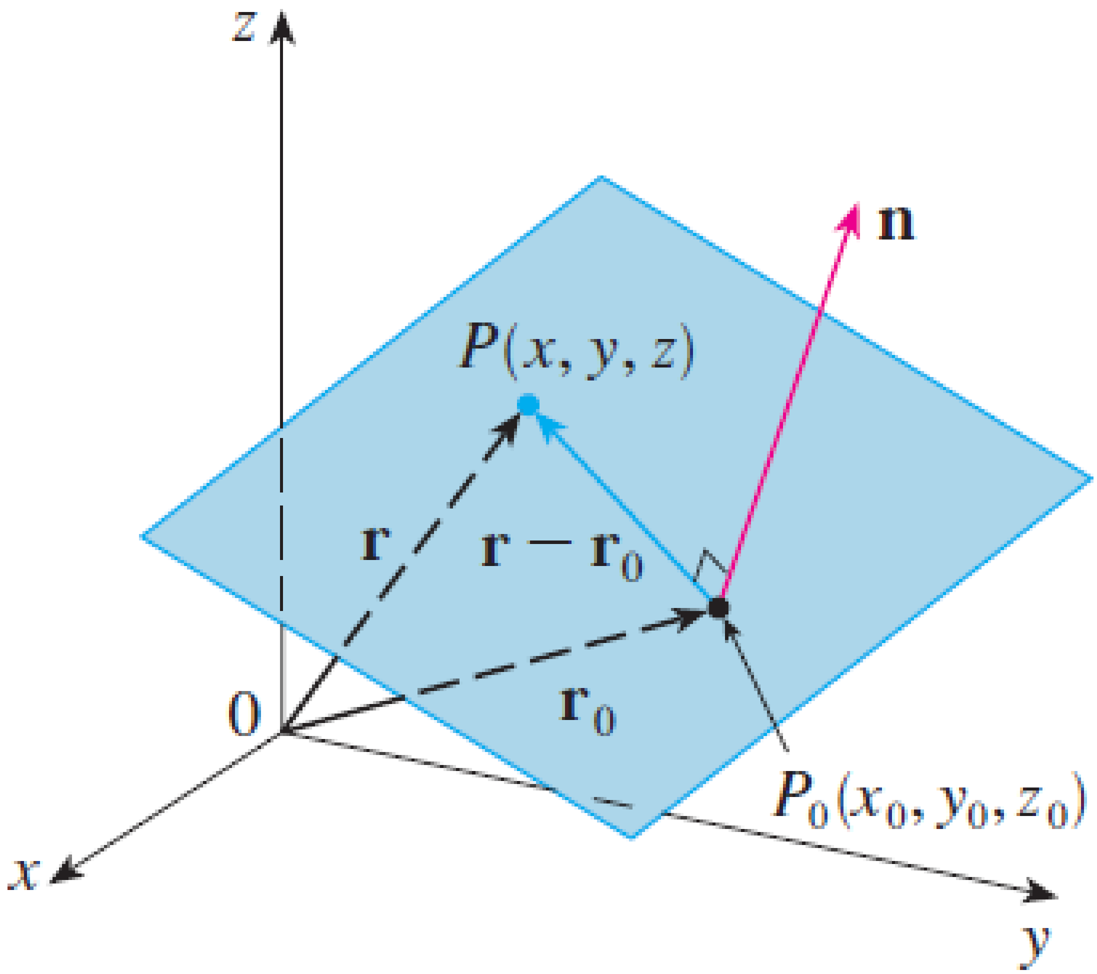
which is sometimes written as:

$$\boldsymbol{r} = \boldsymbol{r_0} + \mathrm{t}\,\boldsymbol{v}$$

Visual of a Line in 3D:



PLANES:



To find the equation for a plane:

$\boldsymbol{n} = \langle a, b, c \rangle =$ a **normal** vector.

$\boldsymbol{r_0} = \langle x_0, y_0, z_0 \rangle =$ a position vector (which means (x_0, y_0, z_0) is a point on the plane)

then if (x,y,z) is any other point on the plane

$$\langle a, b, c \rangle \cdot (\langle x, y, z \rangle - \langle x_0, y_0, z_0 \rangle) = 0,$$

which is sometimes written as

$$\boldsymbol{n} \cdot (\boldsymbol{r} - \boldsymbol{r_0}) = 0$$